

A Directly Extension of Caristi Fixed Point Theorem

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ABSTRACT. In this paper it is proved that if T is a self-map on a complete metric space (X, ρ) and if there exist a lower semicontinuous function $G : \rightarrow \mathbb{R}_+^0$ and an arbitrary fixed integer $k \geq 0$ such that

$$(A) \quad \rho[x, Tx] \leq G(x) - G(Tx) + \dots + G(T^{2k}x) - G(T^{2k+1}x)$$

and $G(T^{2i+1}x) \leq G(T^{2i}x)$ for $i = 0, 1, \dots, k$ and for every $x \in X$, then T has a fixed point ξ in X .

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